

PERFORMANCE OF COOLING-ONLY AND HEATING-ONLY PRISM MODELS IN EXTREME CLIMATES

Michael W. Rufo, XENERGY, Inc., and Michael R. Brambley,
Battelle Pacific Northwest Laboratories, Richland, WA

ABSTRACT

Recently, there has been increased interest in analyzing the usefulness of the Princeton Scorekeeping Method (PRISM) for monitoring consumption in all climates and building types across fuels. Two unique climate applications of PRISM are presented in this paper.

In the first application, the cooling-only PRISM model is used to analyze the electricity consumption of 166 electrically-cooled, gas-heated residences in the hot, humid summer climate of St. Louis, Missouri. In the second application, we use the heating-only PRISM model to analyze the natural-gas consumption of 110 gas-heated residences in the very mild heating climate of San Diego, California. The overall model fit in both applications is reasonably good, but is somewhat poorer than is typically found for gas-heated residences in cold climates. In both cases, the PRISM parameters most poorly estimated are the heating and cooling slopes. In general, the results from the two applications are more similar to each other than to results obtained from applications of PRISM to gas-heated residences in cold climates.

The analysis of the results obtained adds to the small existing information base on the performance of PRISM when applied to air-conditioned residences in humid climates and gas-heated households in very mild winter climates.

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INTRODUCTION

Since its development in the late 1970's, the Princeton Scorekeeping Method (PRISM) has been widely used to model residential energy consumption and provide a statistically-based, standardized measurement of energy savings. Most of the applications of PRISM to date, however, have been to so-called heating-only households, that is, residences for which consumption of the heating fuel is analyzed and for which there is no air conditioning consumption associated with the heating fuel. Also, the fuels analyzed in such studies have generally been those that are easily metered, i.e, natural gas and electricity. In addition, most heating-only PRISM applications have been to households located in cold climates such as New Jersey, Wisconsin, and the Pacific Northwest. These climates are generally characterized by more than 4000 annual base-65 heating degree-days (HDD).

In recent years, there has been an increased interest in ascertaining the efficacy of PRISM in modeling energy consumption for a wider variety of climates and building types and across fuels and space conditioning end uses (Fels, et al., 1986). This paper presents the results of two such novel PRISM applications. In the first application, the cooling-only PRISM model (see Appendix for specifications) is used to analyze the electricity consumption of 166 electrically-cooled, gas-heated households in the hot, humid, summer climate of St. Louis, Missouri. In the second application, we use the heating-only PRISM model to analyze the natural-gas consumption of 110 gas-heated households in the very mild heating climate of San Diego, California.

APPLICATION OF THE COOLING-ONLY MODEL TO HOUSEHOLDS IN ST. LOUIS

Data

The data used for our application of the cooling-only PRISM model to households in St. Louis were originally used in an evaluation of a 1982 residential energy-audit program in Ballwin, Missouri, a small municipality in St. Louis County (Brambley, et al. 1985). The data used in the present study include 12 monthly billing periods of pre- and post-audit electricity consumption (24 data points in all) for January through December 1981 and 1982, respectively, for 166 single-family households with electric central air conditioning and no electric heating. Eighty-one of the households participated in the audit program in January 1982 and comprised the original test group, while the remaining 85 households did not participate in the program and were used as the control group. While the purpose of this paper is not to compare the program-induced savings estimates obtained using PRISM with those found in the original evaluation (this comparison is made in Rufo, 1986), the availability of pre- and post-audit data for both the test and control groups yields a large number of individual observations (332) using the cooling-only model.

Climate

The climate in St. Louis is extremely hot and humid during summer months (June through September). The results in this section will be primarily compared with results obtained in a previous study of cooling-only PRISM applied to 68 households in Lodi, California (Brown, 1986) as well as results that are typical of heating-only PRISM studies. Some comparisons will also be made with results obtained from a study of the cooling-only PRISM model applied to 50 households in New Jersey (Stram and Fels, 1986). The Lodi study was specifically designed as a case study of PRISM in a cooling-intensive climate. It was found, however, that the cooling load in Lodi, which is located in the central valley of California near Stockton, was smaller as a percentage of total annual consumption than that found in New Jersey. As a result, the author called into question the definition of Lodi as a cooling-intensive climate.

One of the principal contributions to the cooling load, cooling degree-days (CDD), is shown in Table I for three different reference temperatures for Stockton, Newark (New Jersey), and St. Louis. Another measure of cooling intensity, the 2.5% design dry-bulb with the mean coincident wet-bulb temperature, is shown for each location in Table II.

Table I. Annual cooling degree-days are shown for varying reference temperatures at selected locations.

Location	Reference Temperature ¹		
	65°F	75°F	80°F
St. Louis	1520	423	142
Stockton ²	1566	386	129
Newark ²	1220	263	63

¹Time periods: St. Louis, 1971-85; Stockton and Newark, 1970-81.
²Source: Brown, 1986.

Table II. The 2.5% dry-bulb and mean coincident wet-bulb temperatures are shown for selected locations.

Location	Design Dry-Bulb/Mean Coincident Wet-Bulb ¹
St. Louis	94/75
Stockton	97/68
Newark	91/73

¹Source: ASHRAE, 1977.

At first glance, the three climates seem similar, however, upon closer inspection it is clear that St. Louis is the most cooling-intensive of the three areas. For example, although St. Louis and Stockton have a similar number of CDD, the mean wet-bulb temperature coincident with the 2.5% design dry-bulb temperature in St. Louis is much higher than that in Stockton.

In addition, the average summertime relative humidity in Stockton is about 50%, whereas that in St. Louis is 75%. Thus, although sensible cooling loads in the two climates may be similar, latent cooling loads in St. Louis are much greater than those in Stockton. When comparing the climate in Newark with that in St. Louis, we can see that although Newark has a high mean coincident wet-bulb temperature, it does not have nearly as many CDD as St. Louis when computed to high reference temperatures. Although St. Louis has only 25% more base-65 CDD than Newark, it has over 200% more base-80 CDD, indicating a greater number of peak cooling days.

PRISM Results

Outlier Identification. After running PRISM for each household for both the pre- and post-audit periods, 31 extreme outlier households (10 from the test and 21 for the control group) were identified. These extreme outliers were characterized by having very large, often infinite, standard errors for α , β , and τ . These households also had very low R^2 s. Visual inspection of the monthly meter readings for these poorly modeled households showed that they had either no discernible summertime load, anomalous data points indicating vacations or data errors, or showed a significant increase in winter consumption that indicated the presence of electric heating. These poorly modeled households were removed from the data set because they did not meet the physical criteria (presence of central air conditioning and absence of electric heating) for the sample. The clean data set thus consisted of 71 test and 64 control residences. The standard errors of the PRISM parameters provided useful flags for identifying problem houses.

Number of Billing Periods to Use. Because PRISM has not been widely used to estimate cooling consumption, the optimal number of billing periods per year to include in the regression is not well established. In the Lodi and New Jersey cooling studies, the number of billing periods selected were 12 and 7, respectively. Including fewer than 12 consumption and weather points might improve the accuracy of the PRISM results because of seasonal variations in non-cooling electricity consumption during winter months attributable to end uses such as water heating, cooking, and lighting. Consumption and weather data for the pre-audit test group were used to compare the results from PRISM using 12 to 8 billing cycles in the regression for households in St. Louis. The results are shown in Table III. Because reducing the number of data points in a regression increases R^2 , the relative standard error of NAC [$se(NAC)/NAC$] is used as a better indicator of the accuracy of the results. Comparing the relative standard errors of NAC in Table III, we conclude that the results become less accurate as winter months are removed from the regression for these data. Therefore, 12 months of billing data are used in our analysis.

Table III. Results from the cooling-only PRISM model with varying winter months removed from the regression are shown for the pre-audit test group (N=71) for the St. Louis cooling data. The relative standard errors of the estimates [i.e., $se(\text{estimate})/\text{estimate}$] are shown in parentheses except for the reference temperature for which the actual standard error is shown. All results are median values.

Number of Months	Period	R ²	Base-Level α (kWh/day)	Cooling Slope β (kWh/°F-day)	Reference Temperature τ (°F)	Normalized Annual Consumption NAC (kWh/year)
12	Jan. to Dec.	0.934	20.0 (0.07)	5.65 (0.36)	72.5 (2.3)	11459 (0.044)
11	Feb. to Dec.	0.940	20.0 (0.08)	5.41 (0.35)	72.0 (2.3)	11396 (0.045)
10	Feb. to Nov.	0.947	19.3 (0.09)	5.26 (0.34)	71.3 (2.3)	11291 (0.045)
9	Mar. to Nov.	0.959	18.3 (0.11)	5.11 (0.32)	70.4 (2.3)	10984 (0.052)
8	Mar. to Oct.	0.960	17.4 (0.13)	4.68 (0.33)	70.0 (2.5)	10930 (0.059)

General Results. The pre- and post-audit results obtained from PRISM for both the test and control groups (using 12 billing periods for each analysis) are shown in Table IV. The relative standard errors of the estimates and R² values for St. Louis are compared in Table V with results obtained in the Lodi cooling study and typical heating-only PRISM results. The median R² values for St. Louis are reasonably high (0.92 to 0.94) and compare well with median values typically obtained in heating-only applications (0.97 to 0.99). The St. Louis median R²s are significantly higher than that obtained in the Lodi study of 0.81 (a median of 0.85 was found in the New Jersey study). Interestingly, the relative standard error of NAC as a percentage of the estimate is higher for St. Louis (4 to 5%) than for Lodi (3%), even though the model fits the data for St. Louis better as measured by R². Even so, the relative standard error of NAC is close to that typically found in heating-only studies (2 to 4%). This is consistent with previous studies of PRISM that show that even when the individual components of the model are poorly determined, NAC is fairly well determined.

Table IV. Results from the cooling-only PRISM model for St. Louis are shown for test (N=71) and control (N=64) groups before and after audits of the test houses. The relative standard errors of the estimates [se (estimate)/estimate] are shown in parentheses for each parameter except the reference temperature for which the actual standard error is shown. All results are median values.

Group	Estimator	R ²	Base-Level α (kWh/day)	Cooling Slope β (kWh/°F-day)	Reference Temperature τ (°F)	Normalized Annual Consumption NAC (kWh/year)
Pre-audit Test	Median	0.934	20.0 (0.07)	5.65 (0.36)	72.5 (2.3)	11459 (0.04)
Post-audit Test	Median	0.934	21.1 (0.06)	8.17 (0.34)	75.0 (1.9)	11572 (0.04)
Pre-audit Control	Median	0.936	22.5 (0.07)	5.37 (0.33)	71.9 (2.2)	12229 (0.04)
Post-audit Control	Median	0.923	22.0 (0.07)	7.23 (0.45)	75.0 (2.3)	11828 (0.05)

Table V. Median accuracy measures of PRISM results are shown for specific cooling-only samples and typical heating-only samples.

Parameter	Cooling-Only		Heating-Only
	St. Louis	Lodi ¹	Typical ²
R ²	0.92-0.94	0.81	0.97-0.99
se(α)/ α	0.06-0.07	0.04	0.10-0.20
se(β)/ β	0.33-0.45	0.46	0.06-0.12
se(τ)	1.90-2.30	2.70	2.00-3.00
se(NAC)/NAC	0.04-0.05	0.03	0.03-0.04

¹Source: Brown, 1986

²Source: Fels, 1986 and Dutt, et al. 1986.

The parameter most poorly determined in this study and the other cooling-only studies is the cooling slope (β). The standard error of the cooling slope as a fraction of the estimate is approximately 33 to 45%. This does not compare well with the 6 to 12% common in heating applications. The Lodi study found similar uncertainty in the cooling slope. These results indicate that the cooling slope is too poorly estimated to be useful for measuring changes in the thermal integrity of the structure from year to year. The poor estimation may be largely attributable to the smaller number of months during which cooling consumption occurs. There are only four months per year during which cooling consumption strongly influences the estimate of the cooling slope, compared with six to eight months of heating-dominated consumption from which heating slopes are usually determined. In addition, the behavioral aspect of air-conditioning (AC) usage increases the variance of the cooling slope because AC systems are often switched on and off throughout the summer. Thus, cooling consumption is not completely thermostatically driven as is implicitly assumed in the linear model.

The one PRISM parameter that seems to have a systematically smaller relative standard error in cooling-only as compared with heating-only applications is α , the estimate of base-level consumption. The median relative standard error of the estimate for α in this study is 6 to 7%, while the value obtained in the Lodi study was even smaller at 4%. These values compare favorably with a range of relative standard errors for α of 10 to 20% generally obtained from heating-only applications. This is not surprising, however, as there are almost twice as many months during which no cooling occurs as those in which there is no heating. Thus, as the estimate of β suffers from too few cooling months, the estimate of α conversely benefits.

One of PRISM's unique features is that it provides estimates not only of base-level and space-conditioning consumption per degree-day, but of the reference temperature of each household as well. The estimated median reference temperatures found for St. Louis varied from 72 to 75°F. The Lodi and New Jersey studies found cooling reference temperatures of similar magnitude. These values are significantly higher than that used to estimate conventional base-65 CDD. These high reference temperatures, which correspond to higher indoor temperatures, may be related to the short length of the cooling season and confounding weather factors, in particular, humidity. If most households do not use air conditioning during non-summer shoulder months, a higher estimate of the reference temperature might improve the overall model fit by reducing the number of cooling degree-days that occur during the shoulder months (April, May, and October in St. Louis) when cooling systems may be shut off. The estimate of cooling consumption [$\beta C_0(\tau)$] for the summer months would not be greatly affected by an increase in τ because β and $C_1(\tau)$ are negatively correlated, i.e., the decrease in $C_0(\tau)$ would be partially offset by an increase in the cooling slope.

Another potentially confounding factor is humidity. The estimated reference temperature is based on average dry-bulb temperatures. Consequently, the use of cooling degree-days as the only explanatory variable in the regression does not explicitly take into account the contribution of the latent cooling load to total cooling consumption. However, we found that addition of a humidity variable (based on absolute as opposed to relative humidity) does not result in reasonable estimates of latent and sensible cooling loads because high absolute humidity levels and high temperatures are strongly correlated when aggregated on a monthly basis. For St. Louis we found a correlation of $r = 0.97$ between monthly average daily dry-bulb temperatures and monthly average absolute humidities for the months May through October. Thus, for St. Louis, average daily temperatures provide a weather pattern consistent with the pattern of absolute humidity levels when aggregated on a monthly basis, as must be done when using consumption data based on monthly billing periods.

Although the separate effects of sensible and latent cooling loads could not be separated for the St. Louis data, the estimated total cooling load does include the contribution of variations in the latent load. This can be seen by comparing the cooling slopes and cooling consumption as a percent of total consumption for St. Louis and Lodi. The median cooling slopes obtained for St. Louis are 2.5 to 3.5 times greater than the value found for Lodi. In addition, cooling consumption as a percent of NAC is approximately 33% for St. Louis compared with only 10% for Lodi (20% for New Jersey). Besides humidity, other factors that may contribute to these differences in relative cooling loads are differences in daily temperature ranges, solar gains, building characteristics (e.g., thermal mass), and air conditioner efficiencies. The mean daily summer temperature range in Stockton is 37°F while the same range in St. Louis is only 21°F (ASHRAE, 1977).

APPLICATION OF THE HEATING-ONLY MODEL TO HOUSEHOLDS IN SAN DIEGO

Data

The San Diego data set consists of 110 gas-heated, single-family residences. These data were originally used to evaluate the energy savings attributable to the installation of low-cost weatherization devices (Brambley, et al. 1984). The test group was comprised of 60 households that had weatherstripping devices installed during the period January through March 1982. The remaining 50 residences made up the control group. The pre- and post-treatment periods used to determine NAC are May 1980 to April 1981 and May 1982 to April 1983, respectively, each with data for 12 monthly billing periods. As mentioned previously, while the purpose of this paper is not to compare the program-induced savings estimates obtained using PRISM with those found in the original evaluation (this comparison is made in Rufo, 1986), the availability of pre- and post-treatment data for both the test and control groups provide a large number of individual observations (220) of the heating-only model applied to households in a very mild heating climate.

Climate

The uniqueness of this application of the heating-only PRISM model is a result of the mildness of the San Diego heating season in comparison with the colder climates that have typified most other PRISM studies. The San Diego houses are located in the transition climatic region of San Diego County. The weather data used are from Gillespie Field, which is in the same region. The relative mildness of the San Diego heating season can be seen in Table VI, which includes annual heating degree-days (HDD) computed using varying reference temperatures for selected locations. The number of base-65 HDD in San Diego (1200) is about one-fourth that of climates with approximately 5000 HDD per year. Moreover, the difference in heating intensity increases dramatically as the heating reference temperature goes up. San Diego has only 87 HDD computed to a reference temperature of 55°F (about 3% of that found in Newark) and 0 base-45 HDD.

Table VI. Annual heating degree-days are shown for varying reference temperatures at selected locations.

Location	Reference Temperature ¹		
	65°F	55°F	45°F
San Diego	1212	87	0
St. Louis	4993	3057	1642
Newark ²	4917	2846	1362
Denver ²	5846	3488	1741
Portland ²	4451	1992	522

¹Time periods: San Diego, 1981-83; St. Louis, 1971-85; Newark, Denver, and Portland, 1970-81.

²Source: Brown, 1986.

PRISM Results

Outlier Identification. After running PRISM for each household for both the pre- and post-treatment periods, 15 extreme outlier households (10 from the test and 5 from the control group) were identified. These households were very poorly modeled and had one or more of the following characteristics: very large or infinite standard errors of the individual estimates, negative heating slopes, and very low R^2 values. Visual inspection of the monthly meter readings for these poorly modeled households showed that they had either no discernible winter heating consumption or anomalous data points indicating vacations or data errors. These poorly modeled households were removed from the data set because they did not meet the physical criteria (presence of natural gas heating and absence of data errors) for the sample. Removing these households resulted in a data set of 50 test and 45 control households. As we found in the cooling-only application, the standard errors of the PRISM parameters provided useful flags for identifying problem houses.

General Results. Median results from PRISM and the relative standard errors of the estimates for the San Diego data are presented in Table VII. The PRISM estimates are, in some cases, somewhat less well-determined than the results usually obtained when PRISM is applied to heating-only households in colder climates (see Table V). The median values of R^2 range from 0.88 to 0.94. The relative standard error of NAC ranges from 4 to 5%. Thus, the overall model fits only slightly worse than typical for cold climate heating-only applications. Interestingly, the overall model fit for the San Diego data, as measured by R^2 and the relative standard error of NAC, is similar to that found in the St. Louis cooling-only study.

Table VII. Results from the heating-only PRISM model for San Diego are shown for the pre-treatment and post-treatment periods for test (N=50) and control N=45) groups for natural-gas consumption. The relative standard errors of the estimates [se (estimate)/estimate] are shown in parentheses for each parameter except the reference temperature for which the actual standard error is shown. All results are median values for each statistical parameter.

Group	R ²	Base Level α (therms/day)	Cooling Slope β (therms/°F-day)	Reference Temperature τ (°F)	Normalized Annual Consumption NAC (therms/year)
Pre-Treatment Test	0.88	0.67 (0.19)	0.18 (0.30)	67.0 (2.4)	566 (0.05)
Post-Treatment Test	0.94	0.66 (0.18)	0.20 (0.20)	65.1 (1.7)	520 (0.04)
Pre-Treatment Control	0.90	0.90 (0.15)	0.21 (0.28)	67.0 (2.3)	662 (0.05)
Post-Treatment Control	0.95	0.90 (0.11)	0.26 (0.19)	64.3 (1.5)	614 (0.04)

As expected, the PRISM parameter most poorly estimated for the San Diego data is the heating slope. The median relative standard error of β ranges from 19 to 30%. This is about three times greater than the relative standard error of β found in cold-climate PRISM studies. The poorer estimate of β is caused by the mildness of the San Diego climate. There are only 5 definite heating months during which consumption is twice as much as monthly consumption during the summer months. Another factor that is a likely contributor to the instability of the heating slope is occupant behavior. It is generally believed that, in mild heating climates, occupants tend to turn their central heating systems on and off throughout the winter. Thus, the same problem that plagues the cooling model affects the heating model in a mild climate: the implicit assumption that space-conditioning is thermostatically driven is violated. The result is that, as was true for the cooling slope estimates, the heating slope for San Diego is too poorly determined to be a useful indicator of temporal changes in thermal integrity.

The accuracy of the estimates of α and τ for San Diego are generally similar to values obtained for other heating-only studies. The relative standard error of α ranges from 0.12 to 0.19, while the standard error of τ is fairly small at 1.5 to 2.4°F.

Overall, the values of R² are generally larger and the relative standard errors of all of the PRISM estimates are smaller for both the test and control groups in the post-treatment heating year. This may be attributable to the fact that there were 30% more HDD in the post-treatment heating year than for the pre-treatment heating year.

CONCLUSIONS

The results from the two applications of PRISM presented in this paper fill what was hitherto a gap in the PRISM literature on the performance of the cooling-only PRISM model in general, and the heating-only PRISM model in mild heating climates. Interestingly, the uncertainty in the results from the two applications are more similar to each other, than either is to uncertainties in the parameters found in previous heating-only studies in cold climates. This is primarily attributable to the instability of β in both the St. Louis and San Diego results. We believe that the uncertainty in the estimate of β common to both applications is a function of the small number of months during which space-conditioning occurs and the fact that occupants may not leave their space-conditioning systems turned on throughout the cooling and (mild) heating seasons.

Despite the uncertainty of β , we have found that the overall model fits reasonably well for both applications as indicated by both the high median R^2 values and the small relative standard errors of NAC. This is consistent with previous PRISM studies, almost all of which show that NAC is generally well-determined even when the individual PRISM parameters that comprise it are not. Thus, depending on the objectives of future studies, PRISM can be successfully used to analyze consumption for households in climates similar to those presented here. However, the values of β obtained from mild heating-only and cooling-only PRISM applications are too uncertain to be useful for analyzing temporal changes in thermal integrity from year to year.

APPENDIX. THE COOLING-ONLY PRISM MODEL

The physical basis of the heating-only PRISM model is well documented elsewhere (Fels, 1986), and, while the cooling-only model is exactly analogous, we present its basic specifications below for the sake of clarity. In the cooling-only PRISM model, energy consumption for an individual household is modeled as a linear function of cooling degree-days per day, i.e.,

$$G_i = \alpha + \beta C_i(\tau) + e_i$$

where G_i is the average daily energy consumption for billing cycle i , $C_i(\tau)$ represents the number of cooling degree-days during billing cycle i , obtained using a reference temperature τ , and e_i is the error for billing cycle i associated with random unexplained variations. The parameter α represents an estimate of annual average base-level consumption per day; β is the constant of proportionality (referred to here as the cooling slope) between the number of cooling degree-days and the weather-dependent (actually, weather correlated) consumption and is a measure of the thermal integrity of the structure; and τ is an estimate of the reference temperature for the residence (that is, the outdoor temperature above which cooling is required). The parameters α and β are estimated by regressing energy consumption (G_i) on cooling degree-days [$C_i(\tau)$]. The value of τ is selected to maximize the coefficient of determination (R^2) for the regression.

In order to remove the effects of weather from estimates of changes in energy consumption, the energy consumption that would have occurred for average weather conditions is estimated using the values of α , β , and τ in the relation

$$\text{NAC} = 365\alpha + \beta C_o(\tau),$$

where NAC is the Normalized Annual Consumption and $C_o(\tau)$ represents the long-term average cooling degree-days per year for the reference temperature τ , averaged over many years.

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