Calculating the Uncertainty of Building Simulation Estimates

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Providing confidence intervals on savings estimates is becoming increasingly important as demand-side management (DSM) impacts grow. System planners want to know how much savings they can count on realizing. Regulators may want to know the probability that a program is not cost-effective. Cost-effective allocation of evaluation resources requires knowledge of the potential reduction in savings-estimate variance relative to the costs of various techniques.

Engineering savings estimates have important functions in DSM planning, program design, and program evaluation; however, they are typically presented as point estimates without confidence intervals. A method of estimating confidence intervals around estimates from engineering algorithms within the context of DSM has been previously presented. Quantifying the uncertainty of engineering models has a long history in the field of instrumentation systems. This paper discusses an application of this approach to engineering simulation estimates.

The analysis of a case study in the paper shows that only a few of the inputs to the estimation of savings contribute significantly to error in the estimate. The results can used be to quantify the value of information being collected, and to redesign data collection to optimize resource expenditures.

Introduction

Calculating confidence intervals on savings estimates is becoming increasingly important as DSM impacts grow larger. System planners want to know how much savings they can count on realizing. Regulators may want to know the probability that a program is not cost-effective. Cost-effective allocation of evaluation resources requires knowledge of the potential reduction in savings-estimate variance relative to the costs of various techniques.

Engineering savings estimates have important functions in DSM planning, program design, and program evaluation; however, they are typically presented as point estimates without confidence intervals. A method of estimating uncertainty around estimates, using error propagation analysis, from engineering algorithms within the context of DSM has been previously presented by Kiefer (1993) and, more generally, by Hummel (1993). Quantifying the uncertain y of engineering models has a long history in the field of instrumentation systems (Dieck 1992). This paper discusses an application of this approach to engineering simulation estimates.

Discussion of Terms

Before going further in this discussion, it is useful to define five concepts related to conducting an error propagation analysis: (1) value of information, (2) cost of information acquisition, (3) propagation error, (4) measurement error, and (5) model error.

Value of information can be defined as function of both the magnitude of the error resulting from imperfect information and the cost of the uncertainty that the error produces. The greater the error or the greater the cost of uncertainty, the more valuable the information. Error propagation analysis is a way to estimate the magnitude of the error. The cost represented by that error has three possible components: (1) the cost of supplying the additional generating capacity necessary to ensure that the peak demand is met because of the uncertainty in the DSM savings estimate, (2) the cost of continuing to fund measures or programs that may not be cost-effective, and (3) the lost opportunity cost of failing to implement measures that might be cost-effective. The use of value of information models in DSM planning is discussed in several reports (Swift 1987; Violette et al. 1992; McRae et al. 1992), although the practical applications of such models have been limited, as pointed out by Hummel (1993).

Cost of information refers to the cost of data acquisition required to reduce the error in the impact estimate. This could be the cost of metering, for example. If the cost of the information acquisition is less than the value of that information, then that information is worth acquiring.

Propagation error is the error in the impact estimate that is a function of individual measurement errors in inputs. It is distinct from model error, which is due to misspecification or omitted terms.

Error propagation analysis provides the basis for comparing the contribution of error of various inputs, and for assessing the variance reduction potential of different data collection techniques.

Measurement error is inherent in developing any type of input. The measurement techniques used to develop simulation model inputs include surveys, audits, and end-use metering. The errors associated with these measurement techniques can be random or systematic. Random errors are associated with both sample-based population estimates and individual estimates.

Random errors that occur when some type of sampling is used to develop a population estimate, as is often the case in residential impact estimates, can be calculated through standard statistical techniques. Sampling error is a function of the standard deviation of the variable of interest and the number of data points in the sample. It is reduced as sample size increases; hence a telephone survey is likely to have smaller sampling error than an on-site survey. Standard errors can be calculated for each variable, based on the number of observations associated with that variance.

Individual measurement errors occur when individual impact estimates are developed. This may be the case in commercial program impact estimates, where estimates are developed on a case-by-case basis. If, for example, an estimate of savings for each participant in a commercial lighting program is based on a questionnaire response to what the hours of lighting are, the random measurement error for each individual is likely to be high. Calculation of the error requires that additional information be used, such as data from a more precise measuring tool be used, such as end-use metering. The standard deviation of the ratio between the less-precise and more-precise estimates for a sample will be the individual measurement error.

Systematic measurement error results from a tendency for an estimate to be biased. Such a bias, in the case of estimates of hours of operation for lighting, for example, could be the result of building owners not accounting for lights being on while the janitorial service cleans the building. This type of error, as with random measurement error for an individual estimate, can be detected by using a better measurement tool such as an on-site audit to measure insulation values rather than a telephone survey, or end-use metering to measure operating hours rather than a participant questionnaire.

Model error is inherent in any type of model, even with perfect information, to the extent that the model does not accurately reflect the response of interest. For example, simple steady-state heat transfer algorithms based on degree-days produce significant errors even with perfect inputs because of the simplified, or missing consideration of dynamic interactions between thermal mass, solar and internal gains, and system and occupancy behavior. More detailed models are likely to have less model error. As mentioned previously, model error is distinct from propagation error, and is beyond the scope of this paper.

Calculation of Propagation Error and Confidence Intervals

The error of an engineering savings estimate is a function of the response of the estimate relative to the input parameters and the error in the input parameters. For an engineering model, assuming that the algorithm is unbiased and the errors are independent, the terms can be combined as shown in Equation (1) (American Society of Heating, Refrigerating, and Air-Conditioning Engineers 1991; Dieck 1992):

$$\sigma_R = \sqrt{\sum_{i=1}^{P} \left(\frac{\partial R}{\partial p_i} \sigma_{p_i} \right)^2} \qquad \textbf{(1)}$$

where: σ = error
R = result (impact estimate)
p = parameter
P = number of parameters.

The partial derivatives shown in this formula can be thought of as the sensitivity of the result with respect to each input. For the engineering estimates based on algorithms such as those used for lighting and motor measures, these partial derivatives are relatively easy to calculate directly.

If a complex simulation model is used, the derivatives must be estimated through parametric runs of the model. Calculating these derivatives is known as “dithering”

(Dieck 1992), and this procedure is used for estimating the propagation error of problems that have no closed form equation. Spitler et al. (1989) discussed the calculation of partial derivatives, or influence coefficients, using the BLAST simulation model. The approach involved perturbing the parameter of interest and dividing the change in the result by the change in the parameter, as shown in Equation (2):

$$IC_{p_i} = \frac{R_{p_i=b+\Delta p} - R_{p_i=b}}{\Delta p} \tag{2}$$

where: IC = influence coefficient, or estimated partial derivative
b = base value of parameter
Δ p = the perturbation.

The units on this derivative are the units of the result divided by units of the parameter. Spitler discusses other forms of influence coefficients, including nondimensional types.

If the result of interest is an energy savings estimate, it is calculated as the difference in two simulation runs: one before the installation of the energy efficiency measure and one after the installation, as shown in Equation (3):

$$IC_{p_i} = \frac{(R_{base} - R_{ee})_{p_i=b+\Delta p} - (R_{base} - R_{ee})_{p_i=b}}{\Delta p} \tag{3}$$

A 1-α confidence interval can be calculated, assuming the savings estimate is a normally distributed variable, by assuming the relationship shown in Equation (4):

$$P(x_1 < X < x_2) = P(z_1 < Z < z_2) = (1 - \alpha) \tag{4}$$

where: X = the actual savings
x_1, x_2 = the lower and upper bounds of the confidence interval
Z = the normal random variable with mean zero and variance 1
z = $(x - \mu)/\sigma$, where μ is the savings estimate.

The confidence interval expressed as a fraction of the mean, $(\mu - x_1)/\mu$ (note that $\mu - x_1 = x_2 - \mu$), therefore is calculated as shown in Equation (5):

$$\frac{\mu - x_1}{\mu} = \frac{z\sigma}{\mu} \tag{5}$$

For a 90% confidence interval, using a two-tailed test, Z = 1.65.

Case Study: Ceiling Insulation Savings

As a case study, we calculated the propagation error and confidence interval in an estimate of residential ceiling insulation annual cooling energy savings in a typical, single-family house with central air conditioning in a warm climate. We modeled the gross savings estimate for improving the ceiling insulation level from no insulation to a nominal R-19 using the DOE-2.ld simulation model (Lawrence Berkeley Laboratories 1992). DOE-2.ld simulates hourly heating and cooling loads imposed on buildings from ambient temperature, solar and internal gains, and other sources, and calculates the energy required for space conditioning to meet these loads and those of other end-uses in the building. The baseline estimate of savings was 823 kWh per year. Error estimates were calculated from audit data. There were three steps in the process: (1) identification of key parameters, (2) estimation of errors in key parameters, and (3) calculation of influence coefficients and propagation error.

Identification of Key Parameters

There are hundreds of inputs in to a simulation model as complex as DOE-2; however, typically only of few of them are significant. Time and resource constraints will usually dictate that calculation of the propagation error and confidence intervals include only the significant variables, Expert judgment is required to select the variables. Although omission of other variables will cause the error estimate to be biased low, the results presented here indicate that such biases are not likely to be significant.

In this ceiling insulation example, we thought that the following inputs might be significant: (1) insulated ceiling area, (2) base case ceiling R-value (including framing and drywall), (3) added ceiling R-value, (4) window area, (5) shading coefficient, (6) glass conductance, (7) wall insulation level, (8) internal loads, (9) cooling system setpoint, (10) cooling capacity, and (11) cooling system efficiency.

Estimation of Errors in Key Parameters

We used audit data as the basis for most of input values and calculation of standard errors in the inputs. Standard errors were calculated as the sample standard deviation of the value. The standard error reflects uncertainty due to both the inherent or natural variation in the parameter of interest and the random error in the measurement tool, i.e., the audit. As was discussed previously, systematic error in the audit procedure cannot be ascertained without the use of some type of more detailed measurement tool. Values for some inputs were not included in the audit data; for these cases we used available point estimates for the inputs and estimated the standard errors using judgement. Table 1 presents the baseline input values, the standard errors, the coefficient of variation, and whether the errors were based on each input.

We also calculated the covariances of inputs, when possible, since a key assumption in the use of Equation (1) is that the errors are unbiased. We did not find any significant covariance between terms.

Calculation of Influence Coefficients and Propagation Error

We calculated the influence coefficient for each key input by perturbing the simulation model over the range of the standard error and using Equation (3). Using Equation (l), and substituting the influence coefficients for the partial derivatives, we calculated the error in the savings estimate. The total error of the estimate is 277 kWh. Using Equation (5), the 90% confidence interval was calculated as shown in Equation (6) below:

$$\frac{z\sigma}{\mu} = \frac{1.65 * 277}{823} = 56\% \qquad (6)$$

We also calculated the contribution each input to the overall error (Cp), as shown in Equation (7) below:

$$C_p = \frac{(IC_p \sigma_p)^2}{\sum_{i=1}^{P} (IC_{p_i} \sigma_{p_i})^2} \qquad (7)$$

Three inputs contributed 90% of the overall error: (1) cooling system efficiency, (2) base case existing ceiling R-value, and (3) insulation ceiling area.

No other input contributed more than 3% to the overall error.

Table 2 presents the influence coefficient for each input, the error in the result, and the contribution of each input.

Discussion

Based on the assumptions made in this case study, uncertainty in the engineering estimate of ceiling insulation is dominated by 3 of the 11 inputs evaluated. Baseline insulation levels and air conditioner efficiency contribute by far the greatest amount to error, and ceiling insulation area was also significant. Efforts to reduce error and increase precision would be most fruitful if they focused on these inputs. Careful recording of baseline insulation levels is often not included in performance

Table 1. Baseline Values and Estimated Errors for Each Key Input

Input	Units	Baseline Value	Standard Error	Coefficient of Variation	Source
Added ceiling insulation	$(ft^2*F*h)/Btu$	17.4	4.71	27%	Audit data
Base case ceiling insulation	$(ft^2*F*h)/Btu$	3.5	1.0	29%	Judgment
Insulated ceiling area	ft^2	1,500	150	10%	Judgment
Window area	% of floor area	15	5	33%	Audit data
Shading coefficient	dimensionless	0.85	0.15	13%	Judgment
Glass conductance (w/o outside air film)	$Btu/(ft^2*F*h)$	1.24	0.18	15%	Audit data
Wall insulation level	$(ft^2*F*h)/Btu$	3.9	0.59	4%	Audit data
Internal loads	$Btu/(ft^2*h)$	1.92	0.38	20%	Judgment
Cooling system setpoint	degrees F	77	3.1	4%	Audit data
Cooling capacity	Btu/h	37,000	8,530	23%	Audit data
Cooling system efficiency	SEER	7.5	1.58	21%	Audit data

Table 2. Calculation of Total Error and Error Contribution for Each Input

Input	Standard Error (SE)	Influence Coefficient (IC)	$(IC*SE)^2$	Contribution
Added ceiling insulation	4.71	6.3	880	1%
Base case ceiling insulation	1.0	-178.5	31,856	42%
Insulated ceiling area	150	0.55	6,706	9%
Window area	5	-845.5	1,787	2%
Shading coefficient	0.15	-100.6	228	0%
Glass conductance	0.18	8.0	2	0%
Wall insulation level	0.59	12.4	53	0%
Internal loads	0.38	-34.9	179	0%
Cooling system setpoint	3.1	-13.8	1,832	2%
Cooling capacity	8.530	0.003	499	1%
Cooling system efficiency	1.58	-114.4	32,660	43%
	Sum	76,686	100%	
	Square root of sum	277		

tracking systems: this analysis indicates the value of doing so. (In this particular case, the baseline insulation level is not likely to be less than the baseline input, since that value is based on the assumption of no insulation, but there may be cases where the baseline insulation is greater than zero.) More audits or field tests to reduce the sampling error associated with air conditioner efficiency may be in order. Extra care should be taken that the tracking of ceiling insulation area is precise. Tracking of other information such as internal gains appears to be significantly less worthwhile.

This type of analysis could be extended to estimate the uncertainty of net impact estimates by using simple algorithms to incorporate free rider, free driver, and takeback estimates (Jacobs et al. 1993). Such an extension would be useful if the technique is being used to help allocate evaluation resources, since those inputs may contribute significantly to the net savings estimate uncertainty.

The cost of collecting such information must be weighed against the value of information gained. If the value of variance-reducing information is known, comparison of the costs of various data collection techniques with the value of information produced by those techniques can be used to determine an evaluation budget. If, for example, the cost of field testing air conditioners substantially outweighs the value of the improved information, such improved information would not be worthwhile, even though it substantially reduces the uncertainty of the estimate.

Conclusions

This paper has presented a technique for calculating the error in a simulation estimate and the confidence interval around the estimate. A simple example was presented. In this example, we found that only three out of 11 inputs contributed significantly to the error. Such findings can be used to help allocate future evaluation resources.

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