Decomposition of Changes in Energy Use: The Comparison of Two Approaches from a Canadian Perspective

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ABSTRACT

Since 1996, the Canadian government has been using factorization analysis to examine trends in industrial energy use. This has greatly improved the scope and accuracy of the analysis. Factorization analysis permits the tracking of trends in energy efficiency both at an aggregate and industry level, which has led to a broadened understanding of energy use in Canadian industry.

Factorization analysis decomposes changes in energy use into three main components: changes in production or output (activity), changes in the mix of sub-sectors (structure), and changes in the amount of energy required for each unit of output in each sub-sector (intensity). Factorization analysis is an important and useful tool used to examine and improve the understanding of changing trends in energy use. This is why numerous types of factorization analysis have been developed, analyzed and used around the world.

Two of the more common factorization methods are the Laspeyres index method and the Divisia index method. The Laspeyres index uses the first year as a weight and has the advantage of being mathematically simple and easy to understand. The Divisia index is the weighted average of relative growth rates and has the advantage of approximating a continuous function. The Divisia index is however more mathematically complex.

The Canadian government is constantly evaluating and improving its analysis of industrial energy use. Although the Canadian government has traditionally used the Laspeyres index in its analyses of trends in industrial energy use, the Divisia index is a very highly supported method, which could further improve Canada's analysis of industrial energy use. This paper compares the Laspeyres and Divisia index methods using detailed Canadian industrial energy use data from 1995 to 2001.

Introduction

Over the last three decades factorization analysis has become a frequently used tool to examine trends in energy efficiency. It allows for the decomposition of energy use into changes in production or output (activity), changes in the mix of sub-sectors (structure), and changes in the amount of energy required for each unit of output in each sub-sector (intensity[1]). Since 1996, the Office of Energy Efficiency (OEE) at Natural Resources Canada has been using the factorization method to produce its *Energy Efficiency Trends in Canada* publication. The use of decomposition analysis has greatly improved the understanding of energy efficiency trends in the Canadian industrial sector.

[1] Although the OEE refers to this effect as "energy efficiency" in its *Energy Efficiency Trends in Canada* publication to avoid confusion with "aggregate intensity" (ie. Total industrial energy divided by total industrial GDP), for purposes of this paper, the term "intensity effect" will be used since it includes a variety of factors including improvements in energy efficiency, weather changes and changes to the mix of products available on the market.

Although the factorization methodology has been internationally agreed upon as a useful tool for energy, energy intensity and greenhouse gas emissions analysis, the best factorization method to use is still up for debate. The two most commonly used methods are the Laspeyres and the Divisia indexes. The Laspeyres index examines energy trends based on percentage changes whereas the Divisia index uses logarithmic growth rates. Even within these general approaches there are many different variations. The general forms of both the Laspeyres and Divisia indexes yield residuals. However numerous methodologies have been developed that allow for a perfect factorization where the change in energy use is fully decomposed between activity, structure and intensity. (Ang & Choi 1997; Padfield 2001; Sun 1996)

Decomposition analysis can generally be examined as either multiplicative or additive. This paper discusses additive changes, meaning:

$$\Delta E_{Tot} = \Delta E_{Act} + \Delta E_{Str} + \Delta E_{Int} \qquad [1]$$

The next section describes how the Laspeyres and Divisia indexes yield the above equation and explain the main advantages and disadvantages of both methodologies. In the third section we present and discuss results using Canadian industrial data for 1995 to 2001.

Methodology

The Laspeyres Index Method

Below we derive the Laspeyres methodology currently used by the OEE. It is based on the following identity:

$$E = A\frac{E}{A} = A\Omega \qquad [2]$$

Denote total energy use in petajoules as E and total industrial activity as A where Ω is "aggregate" energy intensity.

$$xy-1=(x-1)+(y-1)+(x-1)(y-1) \qquad [3]$$

By comparing [2] in the current period to a base year and applying the identity in [3], the change in energy use can be broken down into its activity and "aggregate" energy intensity components, yielding,

$$\frac{E_t}{E_0} - 1 = \left(\frac{A_t}{A_0} - 1\right) + \left(\frac{\Omega_t}{\Omega_0} - 1\right) + \left(\frac{A_t}{A_0} - 1\right)\left(\frac{\Omega_t}{\Omega_0} - 1\right) \qquad [4]$$

Where the final term represents the interaction between activity and "aggregate" energy intensity. "Aggregate" energy intensity itself can be broken down further into structural and intensity components. In doing this, total energy use can be re-expressed where *i* represents each industrial sub-sector:

$$E = A\frac{\sum_i E_i}{A} = A\sum_i \frac{A_i}{A}\frac{E_i}{A_i} = A\sum_i a_i\Omega_i \qquad [5]$$

Where Ω_i is sub-sector i's energy intensity. Comparing energy use in the current period to that in a base period yields an index form:

$$\frac{E_t}{E_0} = \frac{A_t\sum a_{i,t}\Omega_{i,t}}{A_0\sum a_{i,0}\Omega_{i,0}} \qquad [6]$$

Applying [3] in its three-term form to [6] implies,

$$\begin{aligned}\frac{E_t}{E_0}-1 &= \left(\frac{A_t}{A_0}-1\right)+\left(\frac{\sum a_{i,t}\Omega_{i,0}}{\sum a_{i,0}\Omega_{i,0}}-1\right)+\left(\frac{\sum a_{i,0}\Omega_{i,t}}{\sum a_{i,0}\Omega_{i,0}}-1\right)\\ &+\left(\frac{\sum a_{i,t}\Omega_{i,0}}{\sum a_{i,0}\Omega_{i,0}}-1\right)\left(\frac{\sum a_{i,0}\Omega_{i,t}}{\sum a_{i,0}\Omega_{i,0}}-1\right)\\ &+\left(\frac{A_t}{A_0}-1\right)\left(\left(\frac{\sum a_{i,t}\Omega_{i,0}}{\sum a_{i,0}\Omega_{i,0}}-1\right)+\left(\frac{\sum a_{i,0}\Omega_{i,t}}{\sum a_{i,0}\Omega_{i,0}}-1\right)+\left(\frac{\sum a_{i,t}\Omega_{i,0}}{\sum a_{i,0}\Omega_{i,0}}-1\right)\left(\frac{\sum a_{i,0}\Omega_{i,t}}{\sum a_{i,0}\Omega_{i,0}}-1\right)\right)\end{aligned} \qquad [7]$$

For simplicity,

$$\frac{\sum a_{i,t}\Omega_{i,0}}{\sum a_{i,0}\Omega_{i,0}}-1 = \frac{S_t}{S_0}-1 \qquad [8]$$

$$\frac{\sum a_{i,0}\Omega_{i,t}}{\sum a_{i,0}\Omega_{i,0}}-1 = \frac{I_t}{I_0}-1 \qquad [9]$$

$$\left(\frac{\sum a_{i,t}\Omega_{i,0}}{\sum a_{i,0}\Omega_{i,0}}-1\right)\left(\frac{\sum a_{i,0}\Omega_{i,t}}{\sum a_{i,0}\Omega_{i,0}}-1\right)=\delta \qquad [10]$$

$$\left(\frac{A_t}{A_0}-1\right)\left(\left(\frac{\sum a_{i,t}\Omega_{i,0}}{\sum a_{i,0}\Omega_{i,0}}-1\right)+\left(\frac{\sum a_{i,0}\Omega_{i,t}}{\sum a_{i,0}\Omega_{i,0}}-1\right)+\left(\frac{\sum a_{i,t}\Omega_{i,0}}{\sum a_{i,0}\Omega_{i,0}}-1\right)\left(\frac{\sum a_{i,0}\Omega_{i,t}}{\sum a_{i,0}\Omega_{i,0}}-1\right)\right)=\varepsilon \qquad [11]$$

The two final terms in [7] are interaction terms and are denoted by δ and ε. δ measures the interaction between structure and intensity effects, while ε measures the interaction between activity and structure, intensity and δ. ε is equivalent to the interaction term in [4] since structure, intensity and δ make up "aggregate" intensity.

As it has been suggested, (Padfield 2001; Sun 1996) these interaction terms make each individual variable's effect incomplete and can be confusing to the reader. Since any interaction term can be interpreted as the change in energy use that comes from how the two terms affect each other, they can be reallocated to the two variables' impact on energy use. In the *Energy Efficiency Trends in Canada*, the OEE uses Padfield's method to reallocate the interaction terms as follows and present a complete decomposition model.

Total Activity Effect:

$$\frac{A^T}{A_0^T} - 1 = \left(\frac{A_t}{A_0} - 1\right) + \frac{1}{2}\varepsilon \quad [12]$$

Total Structure Effect:

$$\frac{S^T}{S_0^T} - 1 = \left(\frac{S_t}{S_0} - 1\right) + \frac{1}{2}\delta + \frac{1}{4}\varepsilon \quad [13]$$

Total Intensity Effect:

$$\frac{I^T}{I_0^T} - 1 = \left(\frac{I_t}{I_0} - 1\right) + \frac{1}{2}\delta + \frac{1}{4}\varepsilon \quad [14]$$

We have described the Laspeyres approach in index form above. To look at changes as described in [1] we must multiply both sides of the equation by base year energy use (E_0).

$$\Delta E_{Tot} = \Delta E_{Act} + \Delta E_{Str} + \Delta E_{Int}$$

$$\Delta E_{Act} = E_0\left(\frac{A^T}{A_0^T} - 1\right) \quad [15]$$

$$\Delta E_{Str} = E_0\left(\frac{S^T}{S_0^T} - 1\right) \quad [16]$$

$$\Delta E_{Int} = E_0\left(\frac{I^T}{I_0^T} - 1\right) \quad [17]$$

The Laspeyres index has the benefit of being both mathematically sound and easy to understand. The transparency of the method is important when explaining the results to a diverse audience.

The Divisia Index Method

The Divisia Index begins with the same identity in [2] however instead of forming an index by dividing by a base year, we differentiate. Assuming all variables are continuous and are functions of time, we take the derivative of [5] with respect to time.

$$E' = A'\sum_i a_i \Omega_i + A\sum_i a_i' \Omega_i + A\sum_i a_i \Omega'_i \qquad [18]$$

From this we integrate [18] with respect to time from 0 to T. This will give us the change in energy or ΔE_{Tot}.

$$\Delta E_{Tot} = E_t - E_0 = \int_0^t A'\sum_i a_i \Omega_i + \int_0^t A\sum_i a'_i \Omega_i + \int_0^t A\sum_i a_i \Omega'_i \qquad [19]$$

This separates ΔE_{Tot} into the activity, structure and intensity effects. However [19] is expressed as a continuous function and available data are discrete. Because of this, we need to make an approximation to the continuous function. There have been numerous methods used including the Average Mean Divisia (Boyd, Hanson & Sterner 1988). The main problem with Average Mean Divisia is that because we are approximating a continuous function, there is a residual term. A refined Divisia approach was developed using the following logarithmic weight function that does not yield residual terms. (Ang & Choi 1997)

$$L(E_{i,t}, E_{i,0}) = \frac{(E_{i,t} - E_{i,0})}{\ln\left(E_{i,t} / E_{i,0} \right)} \qquad [20]$$

The additive version of the refined Divisia approach described in Ang & Choi 1997 is detailed below.

$$\Delta E_{Tot} = \Delta E_{Act} + \Delta E_{Str} + \Delta E_{Int}$$

$$\Delta E_{Act} = \sum_i L(E_{i,t}, E_{i,0}) \ln\left(A_t / A_0 \right) \qquad [21]$$

$$\Delta E_{str} = \sum_i L(E_{i,t}, E_{i,0}) \ln\left(S_{i,t} / S_{i,0} \right) \qquad [22]$$

$$\Delta E_{\text{int}} = \sum_i L(E_{i,t}, E_{i,0}) \ln\left(I_{i,t} / I_{i,0} \right) \qquad [23]$$

Although slightly more mathematically complex, the Divisia index has some benefits. It is the only index that is symmetric due to its logarithmic properties. (Ang, Zhang & Choi 1998) This means that the reversal of the base year and current year yields equal and opposing results. This is not that case with the general Laspeyres index or other methods based on percentage change. However some perfect decomposition models that use the Laspeyres index have been shown to pass the reversal test, one of these being the Shapely/Sun method. (Ang 2003)

Results

We applied the two above methods to 50 Canadian North American Industry Classification System (NAICS) industries for 1995 to 2001. These industries cover the mining, manufacturing, forestry and construction sectors. The energy use data is from Statistics Canada and the Canadian Industrial Energy End-Use Data and Analysis Centre (CIEEDAC) at Simon Fraser University. The production data was provided by Informetrica Limited and is a combination of physical units, gross output and gross domestic product (GDP).

Although it is impossible to aggregate the physical units of different industries, it was possible to use physical units data by assigning a base year weight to each industry based on its base year share of GDP. When physical units are available this base year share is multiplied by the change in physical units over the period. When physical units are not available, gross output is used and when neither is available GDP is used.

Using physical units as a measure of activity is seen as very practical and intuitive. However for many industries it is not possible to use a physical measure, due to the heterogeneity of products. For example the computer and electronic products industry is extremely diverse. There is no way of aggregating DVD players with mainframe computers since they are such different products.

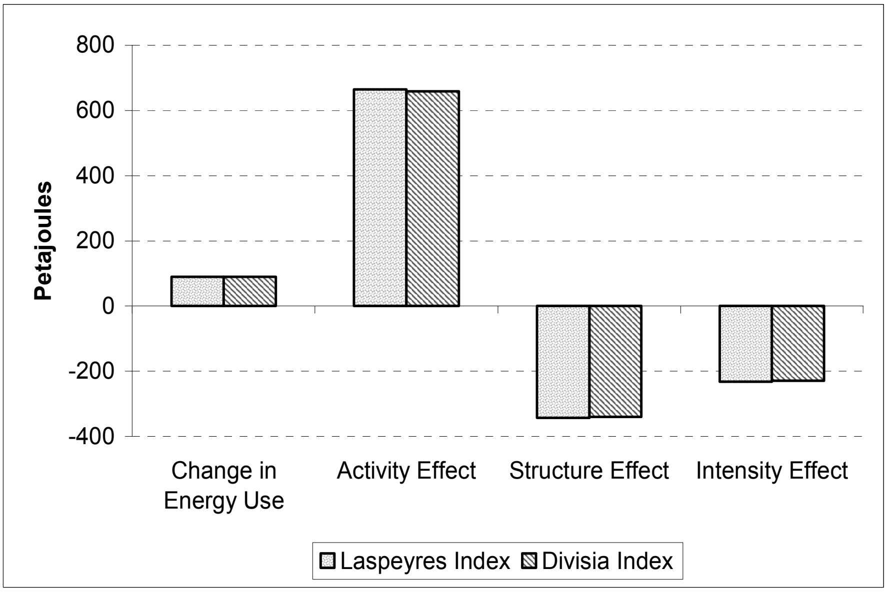
In many cases, value-based activity indicators show quite different trends in growth than do physical units-based measures. Even if you use a constant dollar measure of output, differences can still occur due to price index mismeasurement (Freeman, Niefer and Roop 1997). 1995 to 2001 factorization results with 1995 as a base year are presented in Table 1.

Table 1. Change in Energy Use Attributable to Activity, Structure and Intensity Effects Applied to Canadian Industry: 1995 to 2001 (Petajoules)

		1995	1996	1997	1998	1999	2000	2001
Laspeyres	ΔE_{Tot}	0	83.9	83.6	30.4	124.9	213.6	90.0
	ΔE_{Act}	0	48.8	203.4	343.4	526.6	684.8	664.6
	ΔE_{Str}	0	-34.0	-70.4	-186.0	-235.5	-271.1	-342.8
	ΔE_{Int}	0	69.2	-49.3	-127.0	-166.2	-200.0	-231.8
Divisia	ΔE_{Tot}	0	83.9	83.6	30.4	124.9	213.6	90.0
	ΔE_{Act}	0	48.7	203.3	342.5	524.3	680.7	658.9
	ΔE_{Str}	0	-33.9	-70.4	-185.7	-234.5	-269.4	-339.7
	ΔE_{Int}	0	69.1	-49.2	-126.4	-164.9	-197.7	-229.2

Over the period, both structure and intensity effects helped to avoid a large energy use increase that would have come with the large increases in activity. This is illustrated in Figure 1 where 2001 is compared to the base year of 1995. Structure and intensity's effects on energy use increased over the period. These overall trends are the same whether the Laspeyres or Divisia index is applied.

Figure 1. Impact of Activity, Structure and Intensity on Energy Use: 2001



The differences between the results using the Laspeyres and Divisia indexes are presented in Table 2. The further the current year is from the base year, the larger the difference between the two methods becomes. We also notice this phenomenon in the interaction terms that are generated from the Laspeyres index approach. The reason for this might be due to the assumption that in the Laspeyres methodology, the interaction terms are reallocated equally between the two variables. Although the current methodology does not use a chained index, further analysis will determine the effect that chaining the Laspeyres or Divisia index has on this divergence.

Table 2. The Difference Between the Laspeyres and Divisia Indexes: 1995 to 2001 (Petajoules)

		1995	1996	1997	1998	1999	2000	2001
Difference	ΔE_{Tot}	0	0	0	0	0	0	0
	ΔE_{Act}	0	0.1	0.1	0.9	2.3	4.1	5.7
	ΔE_{Str}	0	-0.1	0.0	-0.3	1.0	-1.7	-3.1
	ΔE_{Int}	0	0.1	-0.1	-0.6	-1.3	-2.3	-2.6

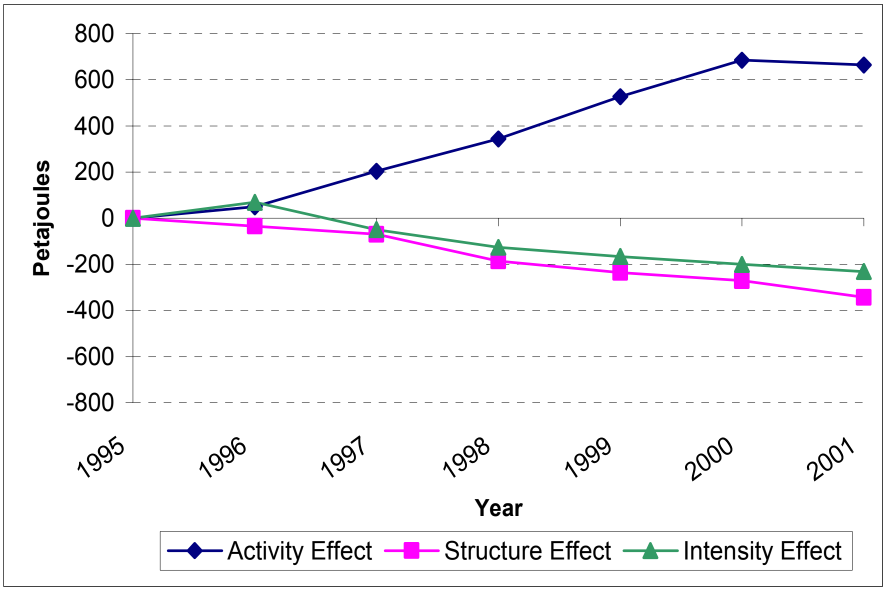
Figure 2 presents the trends in activity, structure and intensity effects over time. Since the results of both approaches are so similar, we present only the Laspeyres method since this is what the OEE currently uses in their analysis.

The activity effect demonstrates that large rates of economic growth would have greatly increased energy use in all years except between 2000 and 2001 where Canadian industry saw a slight economic slowdown. (2% decrease in industrial GDP)

The structure effect measures how a change in industry mix over the period affects energy use. The main driver behind the structural change is the phenomenal growth exhibited by less energy intensive sectors such as the computer and electronics industry. More energy intensive industries such as iron and steel and petroleum refining decreased as a share of total industrial activity.

The intensity effect has also helped to decrease energy use in almost all years. In 1996, energy use increased by 3% while activity increased by less than 2%. Because there was not a large structural impact, energy use would have increased due to an increase in energy intensity. Since 1996, the energy intensity in Canadian industry has improved steadily.

Figure 2. Impact of Activity, Structure and Intensity on Energy Use: 1995 to 2001



Conclusions

As the accuracy of monitoring and tracking of trends in energy use becomes increasingly important, the OEE continues to evaluate and improve on its analytical methodologies. The choice of index method when decomposing changes in energy use rests in the needs of the situation. Over the 1995 to 2001 period, the two methods yield very similar results. However as was illustrated, the longer the period, the greater the difference between the two methods will be. The OEE is committed to presenting very accurate and understandable trends in energy use and energy efficiency. By continually examining and comparing different methodologies, we are able to address both of these needs.

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