Estimating the Remaining Useful Life of Residential Appliances

Cory Welch and Brad Rogers, Navigant Consulting, Inc.

ABSTRACT

Early retirement programs for inefficient equipment can accelerate energy-efficiency savings by increasing the turnover of long-lived technology stocks. Calculating lifetime savings and cost-effectiveness of such programs requires estimating the *remaining useful lifetime* (RUL) of the technology being retired. Estimating an *expected useful lifetime* (EUL) is more common and better understood than estimating an RUL. As a result, RUL estimations are often less accurate and utilize rules of thumb that may not appropriate.

This paper describes a methodology for estimating the RUL of residential technologies using mortality data from surveys combined with Weibull-curve (Weibull 1951) regression analysis. A significant result is that Weibull shape factors for many residential appliances were found to fall within a tight range. The implication is that in the absence of detailed mortality curve data, a single shape factor may be used in conjunction with the more common EUL to estimate the RUL with minimal uncertainty as a function of years in service.

This paper also describes a second method for estimating RUL using a System Dynamics (Sterman 2000) modeling approach with parameter optimization. The model utilizes historical data on appliance shipments and total appliance stock to estimate the mean life and mortality shape factor of the appliance. These parameters can then be used to estimate the RUL in a manner similar to the first approach. Such an approach can be used when neither EUL nor mortality data are available and can also be used to cross-check EUL estimates from standard databases.

Introduction

Early retirement programs for inefficient energy-consuming technologies, such as those currently being implemented by a number of electric utilities through the U.S., can accelerate energy-efficiency savings by increasing the turnover of long-lived technology stocks. Calculating the lifetime savings and cost-effectiveness of such programs requires estimating the *remaining useful lifetime* (RUL) of the technology being retired. Misestimating the RUL can result in mischaracterizing the cost-effectiveness of a program, potentially resulting in overestimation of program benefits and incorrect prioritization of finite energy-efficiency program funding. Estimating an *expected useful lifetime* (EUL) is more common and better understood than estimating an RUL, and a number of sources exist, such as the DEER database (CPUC 2008), that estimate the EUL for many different technologies. RUL estimations often simply employ rules of thumb rather than robust statistical methodologies. For example, one rule of thumb that has been observed is to assume that the RUL of equipment retired early is equal to one third of the EUL. Thus for an EUL of 15 years, the RUL might be assumed to be 5 years when retired early. A more appropriate methodology would take into account the age and mean lifetime of that equipment and would specify the RUL as a function of those parameters.

The impetus for this study was a need to estimate the RUL of residential air conditioning (AC) units in an electric-utility-sponsored energy-efficiency program. The RUL was needed since the utility was required to produce estimates of lifetime energy savings and cost effectiveness of its early retirement program. However, mortality data for residential AC units were not available. Thus, two separate approaches were employed to estimate the likely shape of the mortality curve for residential AC units. The results of this study are important in that they are useful not only for residential AC units, but also for many different residential appliances.

This paper describes two approaches to estimate the RUL as a function of equipment age and equipment EUL. The first approach employs regression of appliance mortality data using a Weibull distribution. The second approach employs System Dynamics to estimate appliance lifetime characteristics from known appliance purchase and total stock data. Although the simulation approach may be less practical than regression of mortality data for future analysis, it can be employed when EUL and mortality data are not available or are suspected to be inaccurate. The second approach is not recommended as standard practice, as it is less practical and somewhat subject to greater error than the first approach if mortality data are available. Nevertheless, the results of applying the second approach strengthen the primary contention in this paper that a common mortality "shape" may be assumed with acceptable levels of uncertainty for many residential appliances.

Approach 1: Weibull Analysis

Mortality data for many technologies tend to follow a pattern that is well described by a Weibull distribution (Weibull 1951), which is commonly used for lifetime analysis (Lawless 1982). Unfortunately, detailed mortality data for residential AC units, the primary technology of interest in this analysis, were not available to permit estimation of a lifetime curve and subsequent estimation of the RUL for a given age of the equipment. However, related technologies often follow a similar *shape* of failure even if their EULs are different. Thus, it was postulated that if data from multiple residential appliances were available, a generic shape factor, or reasonable range of likely shape factors, could be calculated and could facilitate estimation of an RUL for a given EUL and equipment age.

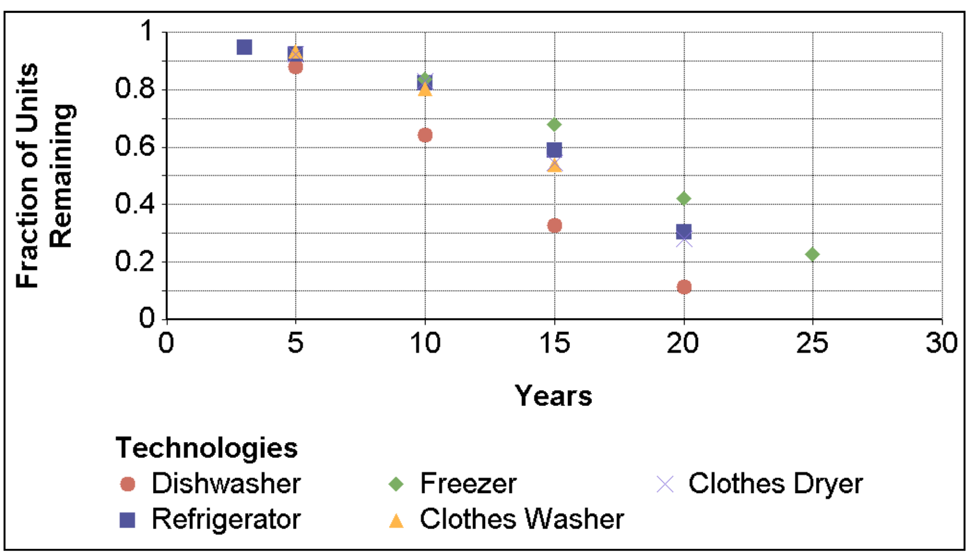
Natural Resources Canada (2003), Energy Efficiency Office, sponsored a Survey of Household Energy Use (SHEU) that collected, among other data, information regarding the age of various appliances at the point of their retirement. Detailed data were gathered for five common residential appliances: dishwashers, refrigerators, freezers, clothes washers, and clothes dryers. These data are provided in the Table 1.

Table 1. SHEU Appliance Mortality Data for Five Appliances

Retirement Age	Instances	Fraction	Cumulative Fraction	Retirement Age	Instances	Fraction	Cumulative Fraction
Clothes Washer Data				**Dishwasher Data**			
< 5 yrs	168,646	0.07	0.07	< 5 yrs	78,987	0.12	0.12
6-10 yrs	323,521	0.13	0.2	6-10 yrs	154,724	0.24	0.36
11-15 yrs	650,238	0.26	0.46	11-15 yrs	206,158	0.32	0.67
16-20 yrs	1,150,056	0.46	0.92	16-20 yrs	140,685	0.22	0.89
> 21 yrs	192,466	0.08	1	> 21 yrs	72,948	0.11	1
Clothes Dryer Data				**Refrigerator Data**			
< 5 yrs	112,794	0.077	0.077	< 3 yrs	221,771	0.05	0.05
6-10 yrs	139,308	0.095	0.172	4-5 yrs	107,860	0.03	0.08
11-15 yrs	416,435	0.284	0.456	6-10 yrs	423,032	0.1	0.18
16-20 yrs	386,880	0.264	0.72	11-15 yrs	1,007,062	0.24	0.41
> 21 yrs	410,668	0.28	1	16-20 yrs	1,206,398	0.28	0.7
				> 21 yrs	1,294,104	0.3	1
Freezer Data							
< 10 yrs	90,810	0.16	0.16				
11-15 yrs	87,372	0.16	0.32				
16-20 yrs	145,186	0.26	0.58				
21-25 yrs	106,914	0.19	0.77				
> 26 yrs	126,841	0.23	1				

Figure 1 illustrates the fraction of appliances remaining as a function of the age of the appliance for the five appliances studied[1].

Figure 1. Appliance Retirement Data



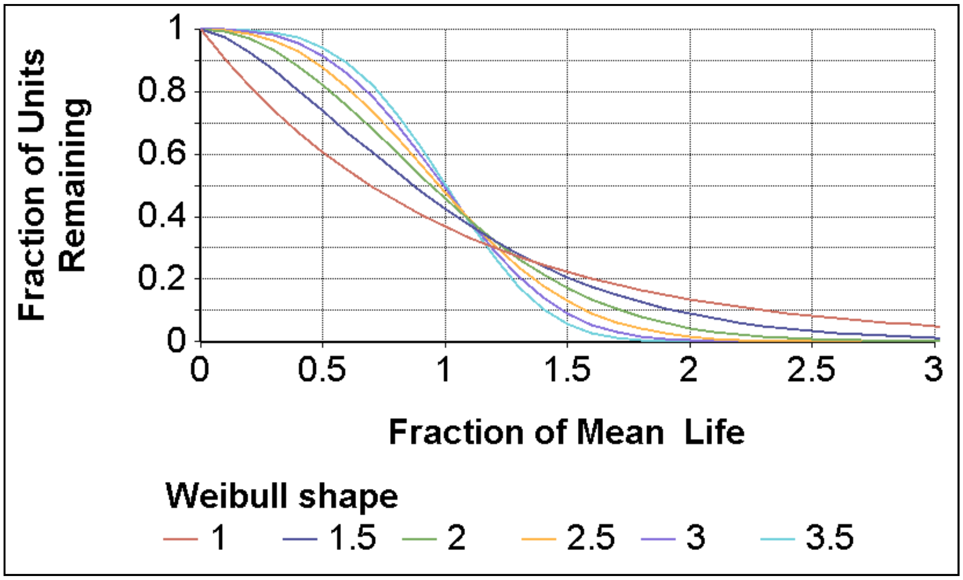
[1] The 20-year data point for clothes washers was discarded as an outlier in this analysis.

To characterize the above failure data, a curve-fit was estimated using the Weibull distribution (Weibull 1951), which has the following characteristics (Wikipedia 2010):

Probability Density Function (PDF) = $(k/\lambda)(x/\lambda)^{k-1}e^{-(x/\lambda)^k}$
Cumulative Distribution Function (CDF) = $1 - e^{-(x/\lambda)^k}$
Fraction of Units Remaining = 1 - CDF
Mean Lifetime (μ) = $\lambda\Gamma(1 + 1/k)$; k = shape factor, λ = scale factor

An illustration of the fraction of units remaining as a function of the fraction of mean life of a technology is provided in Figure 2 for various Weibull shape factors.

Figure 2: Illustration of Various Weibull Shapes on Fraction of Units Remaining



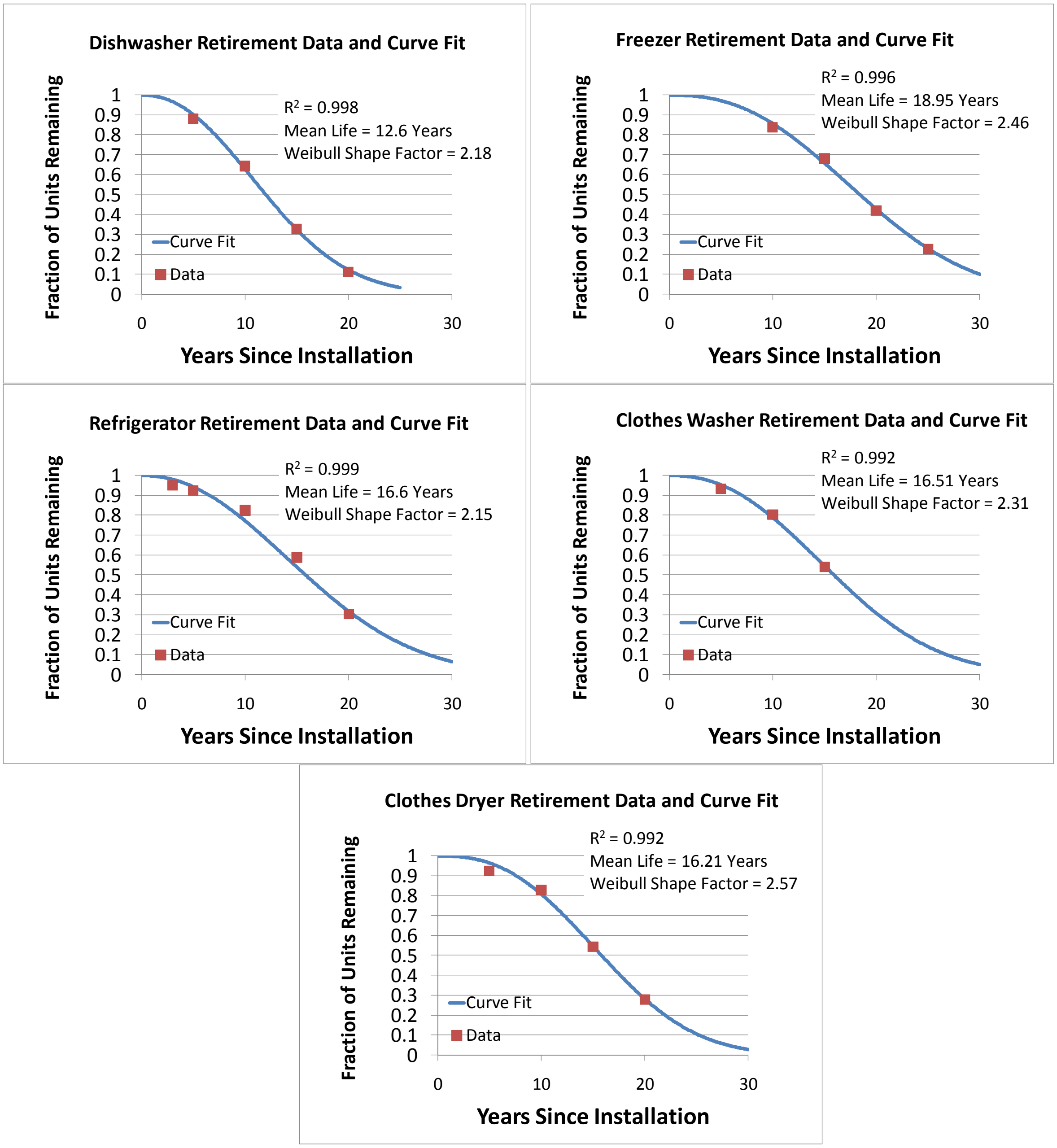
Data for each appliance was fit with a Weibull distribution, adjusting the scale and shape factors using least-squares regression. The regression results are summarized below in Table 2.

Table 2. Parameter Results from the Weibull Regression for Various Appliances

Weibull Regression Results			
Appliance	**Shape factor (k)**	**Scale parameter (λ)**	**Mean Life (μ)**
Dishwasher	2.18	14.22	12.59
Refrigerator	2.15	18.76	16.62
Freezer	2.46	21.36	18.95
Clothes Washer	2.31	18.63	16.51
Clothes Dryer	2.57	18.26	16.21
Average	***2.34***		

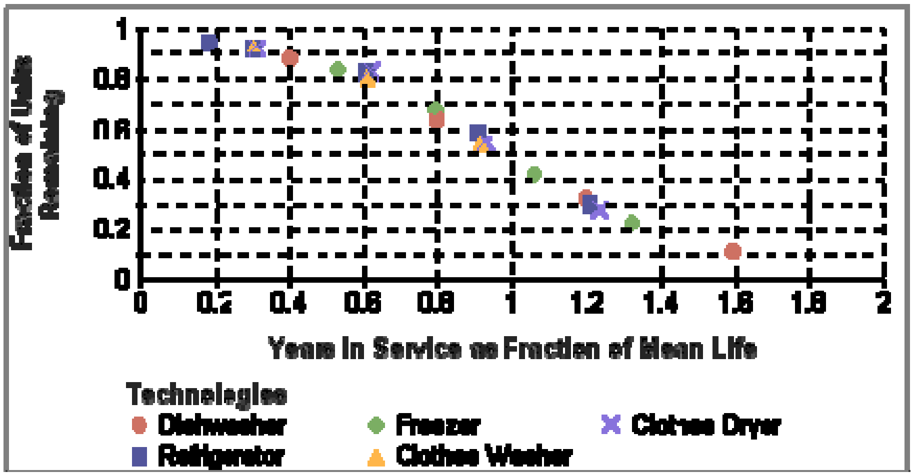
Figure 3 below shows the Weibull curves fit to each set of appliance mortality data.

Figure 3. Curve-Fit of Appliance Mortality Data using a Weibull Distribution



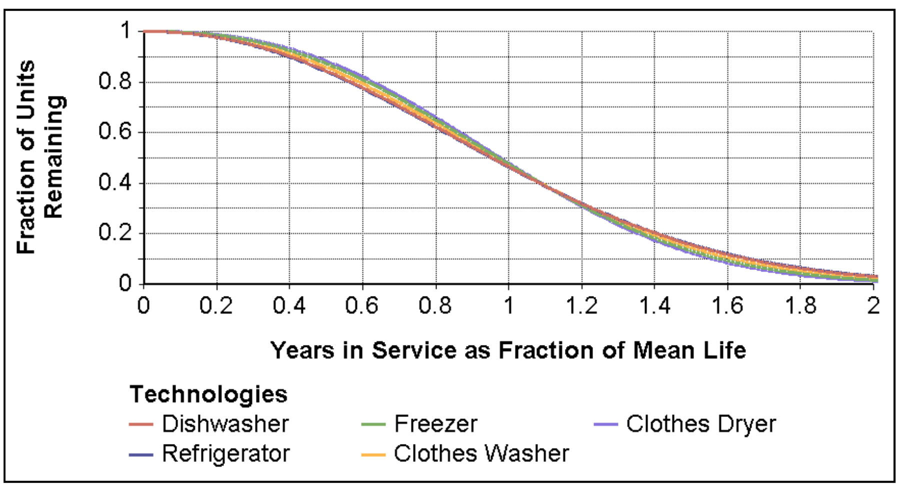
As seen in Figure 3, the Weibull distribution appears to be a good choice curve-fitting to appliance retirement data. The data were then normalized by mean lifetime to illustrate the similarity in the *shape factor* of the mortality curves across these appliances. Figure 4 shows that appliance mortality behaves in a very consistent manner across multiple residential appliances, when normalized by mean lifetime.

Figure 4. Appliance Retirement Data Normalized as a Fraction of Appliance Mean Life



When the normalized appliance mortality data are fit to a Weibull distribution, the outcome is a tightly constrained range of shape factors across appliance types. Figure 5 shows the tight range of Weibull curves fit to the normalized data.

Figure 5. Normalized Weibull Curve Fit to Appliance Data



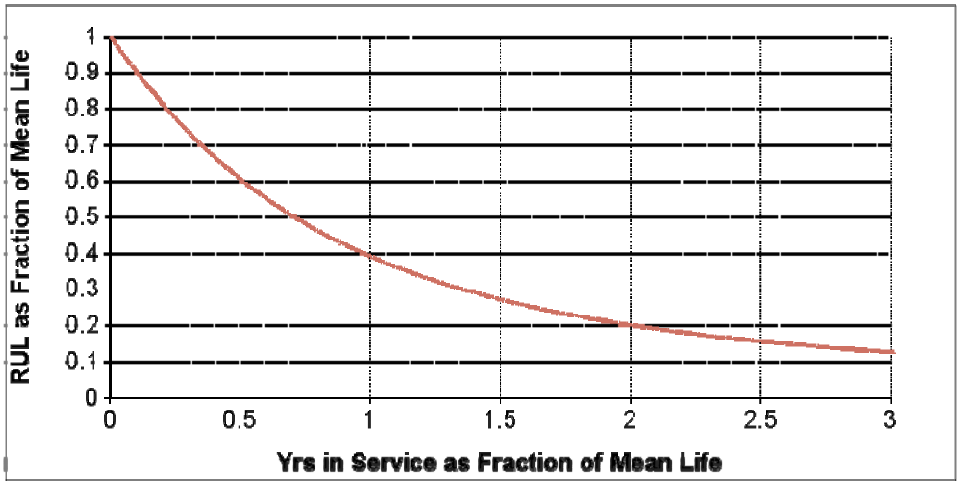
The implication of Figure 5 is that a common, or average, shape factor might be assumed, with a reasonable level of uncertainty, for residential appliance retirement estimation given data regarding the EUL of a particular appliance. Therefore, in this analysis, the average shape factor of 2.34 was selected as that which reasonably characterizes residential appliances with an acceptable level of uncertainty. This value will later be compared with a shape factor estimated specifically for residential central AC units using a different approach.

With an assumed shape factor of 2.34 for the Weibull distribution, one can calculate using numerical methods[2] the remaining useful life for an appliance as a function of the years the

[2] A numerical methods approach was chosen for simplicity, as compared with attempting to derive an analytical solution to the RUL equation for a Weibull distribution.

appliance has been in service. This result is illustrated below in Figure 6 for a shape factor of 2.34 with both the x- and y-axes normalized by the mean lifetime of the technology.

Figure 6: RUL vs Years in Service (Weibull Shape Factor = 2.34)



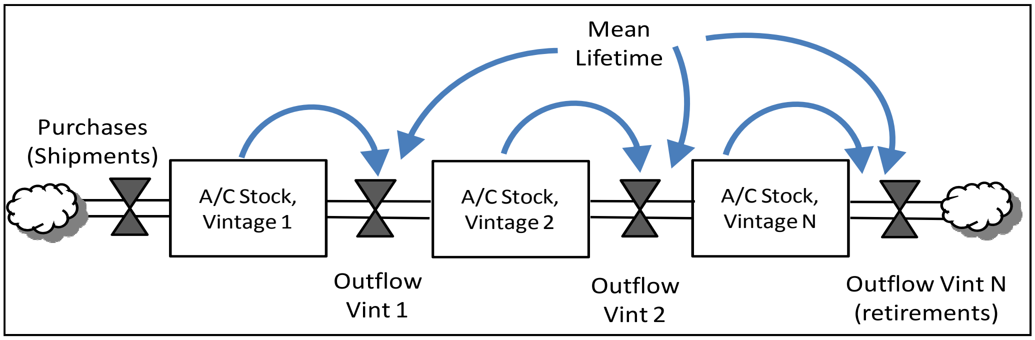
Using Figure 6, one can see for instance that if a product has an EUL of 10 years and has been in service for 10 years, one could expect the product to last for an additional 3.9 years (or 0.39 x 10 years, where 0.39 is the y-axis value read at the x-axis value of 10/10 = 1). Likewise, if the product were in service for 20 years (or 2 x EUL), one could expect the product to last an additional 2 years (or 0.2 x EUL), on average.

A well-designed early retirement program would require collecting information regarding the age of the equipment being replaced to use the above described technique for estimating RUL. If the program is poorly designed and does not capture these data, the evaluation plan should include steps to estimate the life of equipment being replaced via other methods (e.g., telephone surveys of program participants). However, such methods can be expected to be less precise than collection of these data by the implementation contractor at the time of equipment replacement. Once an estimate of the RUL is calculated, it is then relatively straightforward to calculate the lifetime savings and cost-effectiveness of the early retirement program. One could also use this approach to estimate lifetime savings for program planning purposes, although such *ex ante* savings estimates would be less reliable than *ex post* savings calculations since an assumption regarding the average age of equipment to be replaced would have to be made in advance of collecting such data.

Approach 2: System Dynamics Modeling of Technology Stocks

As mortality data were not available for residential central AC units, the technology of interest to the study team, a System Dynamics (Sterman 2000) approach was employed to estimate the EUL and the mortality shape factor of residential central AC units. The System Dynamics approach permits modeling the stock of a technology as a function of the inflows (i.e., new purchases) and outflows (i.e., retirements) of that technology using numerical integration techniques. The retirement (outflow) of units was simulated using an aging chain with each stock exponentially decaying into the downstream stock (Sterman 2000, 470). The last outflow in the chain represents the retirement of AC units. A conceptual illustration of the stock/flow model is provided in Figure 7.

Figure 7. Conceptual Illustration of Stock Aging Chain



Each variable in the figure above is defined below:

Purchases [AC Units/Year]: this is the inflow into the aging chain and was estimated using AC unit shipment data

AC Stock, Vintage $_n$ [AC Units]: = $\int (Inflow_n - Outflow_n)dt$, where

Inflow, Vintage $_n$ [AC Units/Year]: = IF n = 1 THEN Purchases, ELSE Outflow, Vint $_{n-1}$

Outflow, Vintage $_n$ [AC Units/Year]: = AC Stock, Vintage $_n$ / (Mean Lifetime/N)

N = Number of stocks in the aging chain (equivalent to the Erlang "shape factor")

dt = the time step for the numerical integration, in this case 0.25 years

This approach was feasible since data were available regarding the shipment of AC units into the U.S. (AHRI 2009), a proxy for the inflow into the stock aging chain, and data were available that permitted estimating the total stock of AC units in the U.S. over a 27-year period (EIA 2005). The aging-chain approach is analytically equivalent to simulating retirements with an Erlang[3,4] distribution (Sterman 2000, 465), which is similar in shape to the Weibull distribution discussed earlier. The mean life and number of stocks, N, in the aging chain (which is equivalent to the Erlang shape factor) were optimized in the Analytica™ software platform by minimizing the sum of the differences between the AC unit stock *data* and the *simulated* total stock of AC units. Historic growth rates were used to allocate the estimated total AC stock into each of the vintages to ensure the simulation started in dynamic equilibrium (Sterman 2000,

[3] The Erlang distribution aligns with the system dynamic approach to modeling aging chains as an integer (k) number of separate stocks. The Erlang distribution is a gamma distribution that produces exponential type decay (like the Weibull distribution), but where the shape factor is constrained only to integers.

[4] PDF $= \lambda^k x^{k-1} e^{-\lambda x}/(k-1)!$; CDF $= \gamma(k, \lambda x)/(k-1)!$; $\mu(mean\ life) = k/\lambda$; k = shape factor, λ = scale factor

232).[5] Table 3 shows the shipment and stock data that were used as model input variables (shipments) or in the optimization objective function (stock data). As will be subsequently discussed, retirement data (calculated from shipment and stock data) were used as a cross-check against simulated retirements, but were not directly used in the optimization.

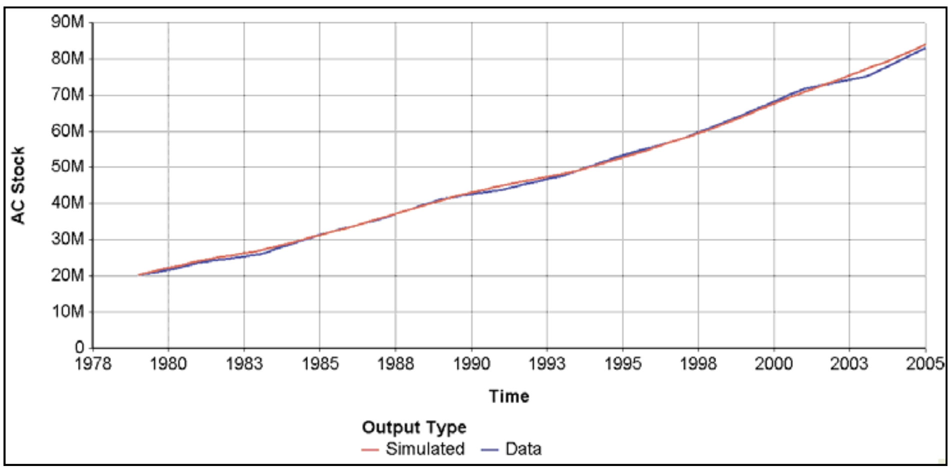
Table 3. Air Conditioner Stock & Flow Data (in Millions of Units)

Year	**1978**	**1979**	**1980**	**1981**	**1982**	**1983**	**1984**	**1985**	**1986**	**1987**	**1988**	**1989**	**1990**	**1991**
AC Shipments	2.7	2.7	2.2	2.4	1.9	2.7	3.3	3.3	3.6	4.0	4.0	4.3	3.7	3.8
AC Stock	23.3	25.0	26.4	28.8	29.9	31.1	34.1	37.1	39.3	41.6	44.5	47.3	48.4	49.6
AC Retirements	0.9	1.0	0.7	0.0	0.7	1.5	0.3	0.3	1.3	1.7	1.2	1.5	2.6	2.7

Year	**1992**	**1993**	**1994**	**1995**	**1996**	**1997**	**1998**	**1999**	**2000**	**2001**	**2002**	**2003**	**2004**	**2005**
AC Shipments	3.7	4.1	4.9	5.1	5.7	5.4	6.2	6.6	6.7	6.3	6.7	6.8	7.4	8.6
AC Stock	51.4	53.2	55.9	58.7	60.8	62.9	66.0	69.1	72.4	75.7	76.9	78.0	81.6	85.3
AC Retirements	1.9	2.3	2.1	2.3	3.6	3.3	3.1	3.5	3.4	3.0	5.6	5.7	3.7	4.9

As illustrated below in Figure 8, an excellent fit of the AC stock data and simulated AC stock was obtained with an EUL (mean lifetime) of 15.46 years and N = 5 (the number of stocks in the aging chain, also the Erlang shape factor). For comparison, the value for mean life of residential central air conditioners used in the DEER database (CPUC 2008) is 15 years, a close match.

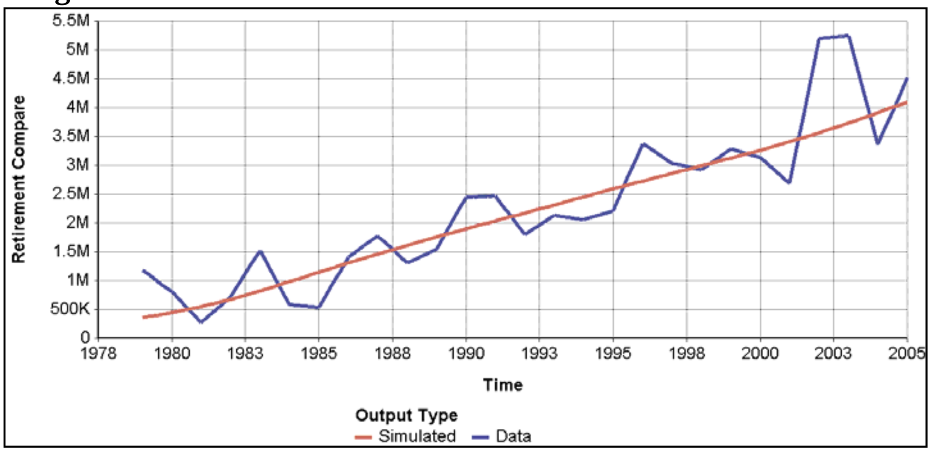
Figure 8: Fit of Simulated Stock to Historical Stock of AC Units



[5] Historic growth rate at the start of the simulation was estimated to be 22%. The allocation of the initial total stock into each stock vintage was calculated according to the following: allocation_fraction $_n$ = $Factor_n / \sum_{n=1}^{N} Factor_n$, where Factor $_n$ = 1/(1+Hist_grwth_rat * μ /N)^(n-1)*(n<=N), n = vintage number, N = total stocks in the aging chain, μ = mean lifetime. This allocation ensures that the simulation starts in *dynamic equilibrium*. The allocation formula, derived by the author, is beyond the scope of this paper.

Although the AC unit retirements were not used in the objective function for optimizing the parameters (only the total stock values, simulated vs. data, were used in the optimization), a comparison was made between the simulated retirements of AC units and the calculated retirements of AC units based on the AC unit stock and shipment data as a cross-check of the modeling approach. While the retirement data are quite "noisy", in general there is good agreement between simulated retirements and retirements estimated from historical stock and shipment data, as illustrated in Figure 9.[6]

Figure 9. Fit of Simulated Retirements to Historical Retirements

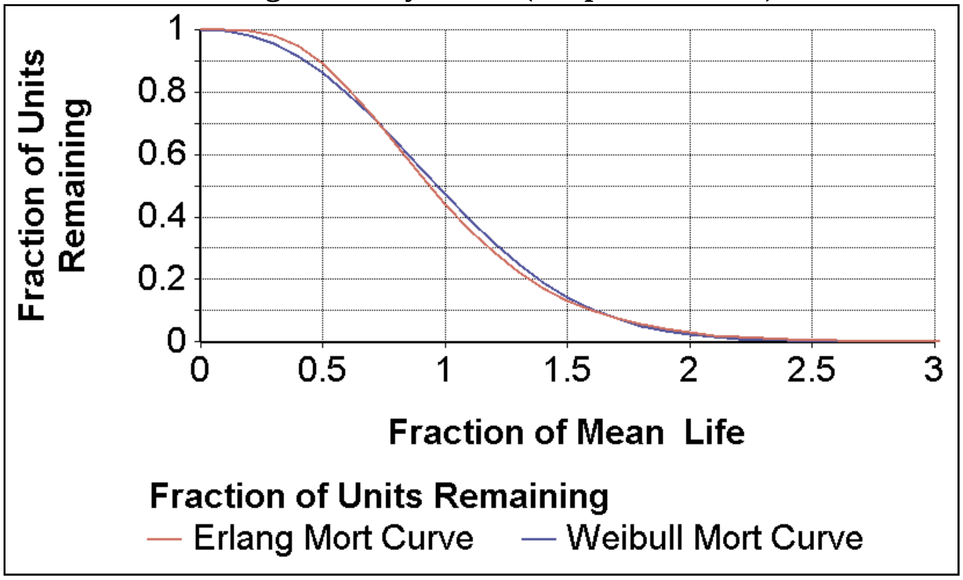


Conclusion

For direct comparison with the first regression analysis method, the optimized Erlang shape factor of 5 is compared with the average Weibull shape factor of 2.34 calculated in the previous section. As can be seen in Figure 10, these mortality curves are quite similar in shape.

[6] The retirement data are noisy largely due to having to calculate retirements from the available stock data and shipment data over time. In other words, retirement "data" were not observed directly.

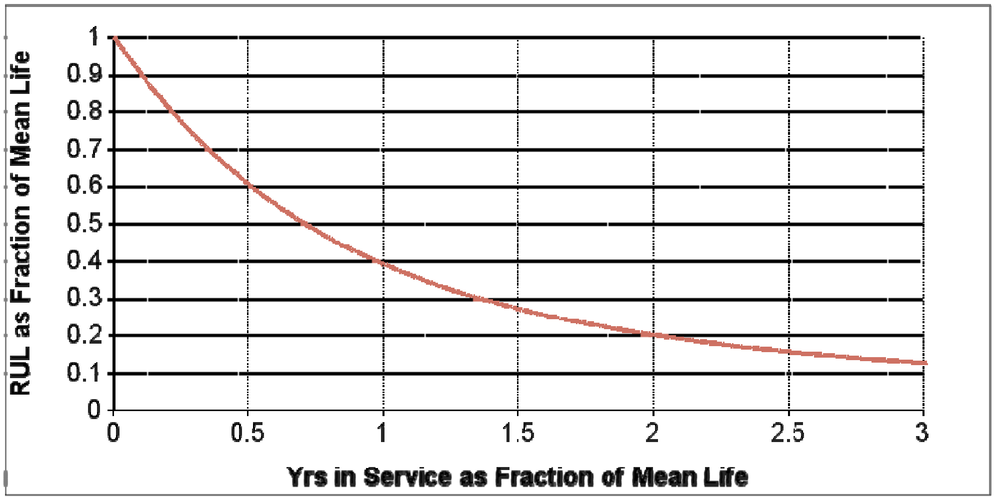
Figure 10: Comparison of Weibull Mortality Curve (Shape Factor = 2.34) and an Erlang Mortality Curve (Shape Factor = 5)



The above figure indicates that a second, independent, method for estimating the retirement shape factor yielded very similar results to the first method for estimating the shape factor for a number of residential appliances. The second approach is not recommended as standard practice, as it is less practical and somewhat subject to greater error than the first approach if mortality data are available. Nevertheless, the results of applying the second approach strengthen the contention that a common Weibull shape factor can be used to reasonably estimate the RUL of a residential appliance given a particular EUL and years in service of the appliance. The "aging chain" simulation method of the second approach is useful elsewhere, however. For instance, it is an elegant and reasonably accurate way of simulating the turnover of the stock of inefficient equipment (as opposed to assuming a fixed lifetime or assuming a pure exponential decay of the stock, as is commonly done in demand side management potential models).

Thus, for the purpose of calculating an RUL for residential appliances without mortality data, a Weibull distribution with a shape factor of 2.34 (the average calculated previously) could reasonably be assumed for purposes of estimating the RUL as a function of EUL and the years in service of the equipment, as shown in Figure 11. Further, our contention is that the same curve can be used for a wide range of residential appliances given estimates of their mean lifetime, or EUL, a parameter commonly found in many technology databases.

Figure 11. Normalized RUL vs. Years in Service (Weibull Shape Factor = 2.34)



References

Natural Resources Canada, Office of Energy Efficiency. 2003. "Survey of Household Energy Use (SHEU), Detailed Statistical Report." http://oee.nrcan.gc.ca/publications/statistics/sheu03/pdf/sheu03.pdf

[AHRI] Air-Conditioning, Heating, and Refrigeration Institute. 2009. Equipment statistics, Central Air Conditioners and Air Source Heat Pumps, Domestic 10/21/2009. http://www.ahrinet.org/Content/CentralAirConditionersandAirSourceHeatPumps_604.aspx

[CPUC] California Public Utility Commission 2008. Database for Energy Efficient Resources. http://www.deeresources.com/

[EIA] Energy Information Administration. 2005. Residential Energy Consumption Survey (RECS), Housing Characteristics. http://www.eia.doe.gov/emeu/recs/recs2005/hc2005_tables/detailed_tables2005.html

Lawless, Jerald F. ed. 2, 2003. *Statistical Models and Methods for Lifetime Data.* Hoboken, New Jersey. Wiley.

Sterman, John D. 2000. *Business Dynamics: Systems Thinking and Modeling for a Complex World.* Boston, Mass.. Irwin McGraw-Hill.

Weibull, W. 1951. “A statistical distribution function of wide applicability.” J. Appl. Mech.-Trans. ASME 18(3), 293-297

Wikipedia 2010. *Weibull distribution.* http://en.wikipedia.org/wiki/Weibull_distribution